Physics 200 68 Name KEY

Final Exam

December 16, 2009

This is a closed book examination. You may use a small 3x5 card with equations on it. There is extra scratch paper available. Explanations must be included with all answers – even multiple-choice questions. Your explanation is worth 75% of the possible points.

A general reminder about problem solving:

- Focus
  - Draw a picture of the problem
  - What is the question? What do you want to know?
  - List known and unknown quantities
  - List assumptions
- Physics
  - Determine approach – What physics principles will you use?
  - Pick a coordinate system
  - Simplify picture to a schematic (if needed)
- Plan
  - Divide problem into sub-problems
    - Modify schematic and coordinate system (if needed)
  - Write general equations
- Execute
  - Write equations with variables
    - Do you have sufficient equations to determine your unknowns?
  - Simplify and solve
- Evaluate
  - Check units
  - Why is answer reasonable?
  - Check limiting cases!
- Show all work!

Possibly useful mathematical relationships:

Law of Cosines $c^2 = a^2 + b^2 - 2ab\cos(\theta)$ which for θ=90° is the Pythagorean theorem $c^2 = a^2 + b^2$

Trigonometric identities:

$$\sin^2(\theta) + \cos^2(\theta) = 1$$
$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$
$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$$

Derivative $\frac{d}{du}Cu^n = nCu^{n-1}$ and anti-derivative (integral) $\int Cu^n du = \frac{1}{n+1}Cu^{n+1} + const.$ of a polynomial

Derivative $\frac{d}{du}k\sin(au) = ka\cos(au)$ and integral $\int k\sin(au)du = -\frac{k}{a}\cos(au) + const.$ of the sine function

Derivative $\frac{d}{du}k\cos(au) = -ka\sin(au)$ and integral $\int k\cos(au)du = \frac{k}{a}\sin(au) + const.$ of the cosine function

The Chain Rule $\frac{d}{dz}f(u) = \frac{d}{dz}u\frac{d}{du}f(u)$

Useful Data:
Mass of Earth, $M_E = 5.97\times10^{24}$ kg
Radius of the Earth, $R_E = 6.38\times10^{6}$ m
Gravitational Constant, $G = 6.67\times10^{-11}$ Nm^2/kg^2

The next two questions involve an experiment involving a very fast vehicle and an annoying buzzer that emits a tone at a frequency f_o. Both vehicles can travel faster than the speed of sound, v_s.

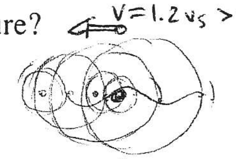
1) [4 pts] The buzzer is stationary and you move away at a speed $v=1.2\ v_s$. What frequency do you hear?
   a) You hear a higher frequency, $f > f_o$.
   b) You hear the same frequency, $f = f_o$.
   c) You hear a lower frequency, $f < f_o$.
   d) You don't hear anything.

$v_s \rightarrow$ $\quad V = 1.2 v_s > v_s$

BUZZER $\quad$ You

Sound waves do not reach you since you are moving faster than the sound waves

2) [4 pts] You are stationary and the buzzer, mounted on the back of the vehicle, moves away at a speed $v=1.2\ v_s$. What wavelength do you measure? $V = 1.2 v_s > v_s$
   a) You measure a longer wavelength, $\lambda > \lambda_o$.
   b) You measure the same wavelength, $\lambda = \lambda_o$.
   c) You measure a shorter wavelength, $\lambda < \lambda_o$.
   d) You don't measure any wave.



You

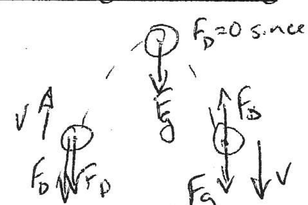
$V=0$

$\lambda > \lambda_o$

$v_s = \lambda f$ for sound in air @ STP

frequency decreases
wavelength increases

3) [4 pts] You throw a tennis ball straight up into the air. Do not ignore air drag. What is the acceleration at the very top of the ball's trajectory?
   a) The acceleration is less than g.
   b) The acceleration is equal to g.
   c) The acceleration is equal to $0 \, m/s^2$.
   d) The acceleration is greater than g.
   e) The acceleration depends on the initial velocity.
   f) Not enough information given.



$\Sigma F = ma$

$F_g = mg = ma$

$g = a$

Acceleration would be g everywhere if there was no drag

4) [8 pts] You have a ring and a solid disk that both have the same radius that are free to rotate (spin) about their axis. The moment of inertia of the disk ($\frac{1}{2}mr^2$) and the moment of inertia of the ring (mr^2) are equal since the disk has twice the mass of the ring. If you apply the same torque to these objects for the same distance, which object has a greater angular velocity? Assume both objects start from rest.
   a) The ring
   b) The disk
   c) They both spin at the same rate

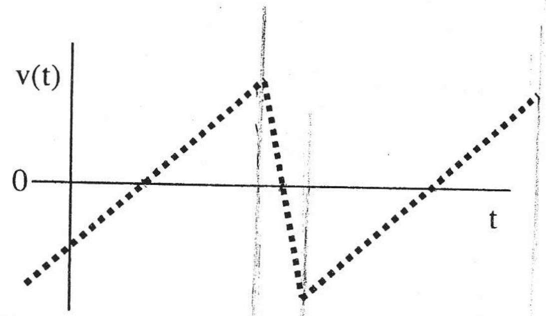
$\int \vec{\tau} \cdot d\vec{\theta} = \Delta E$

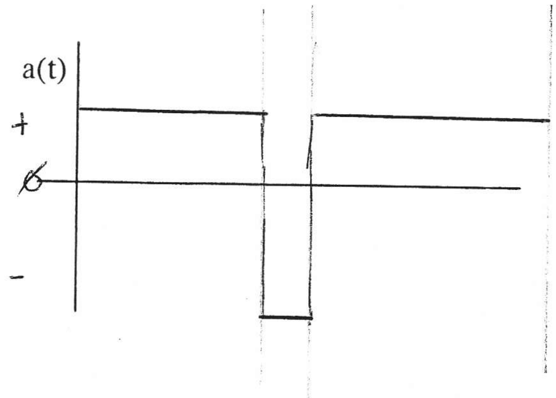
Both objects have the same KE after being spun

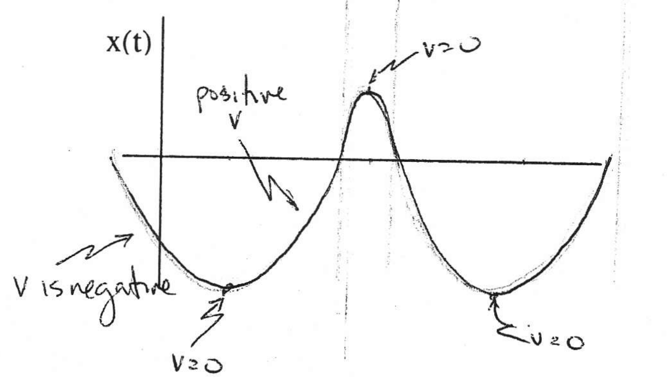
$\therefore KE_D = KE_R$

$\frac{1}{2} I_D \omega_D^2 = \frac{1}{2} I_R \omega_R^2 \quad I_R = I_D \therefore \omega_D = \omega_R$

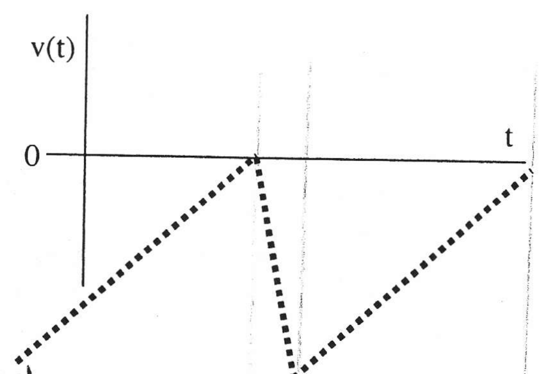
5) [8 PTS] You observe an object with the following velocity as a function of time graph. Draw the corresponding position and acceleration graphs?

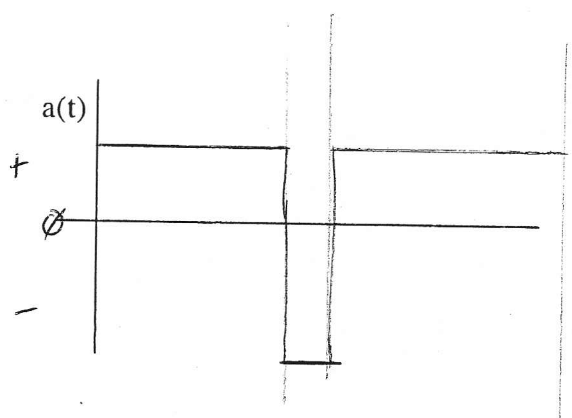


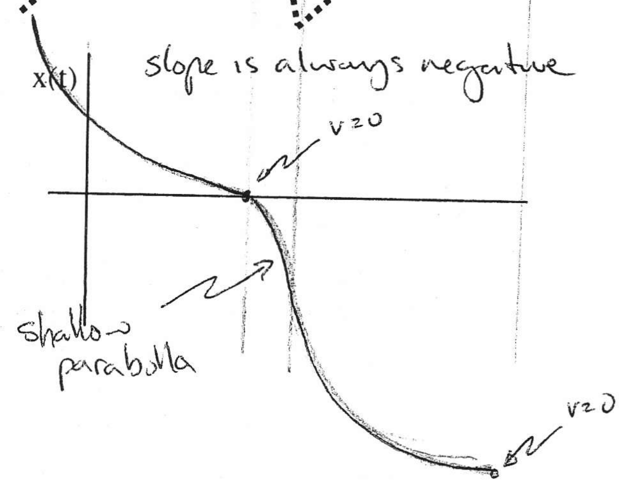




6) [8 PTS] You observe another object with a very similar shaped velocity as a function of time graph. Draw the corresponding position and acceleration graphs. Explain what is the same and what is different about these two v(t) graphs.







They have the same a(t) graphs – while the shape of their v(t) graphs are similar – the resulting x(t) graphs are quite different

7) [4 pts] You have made a pendulum (a large mass suspended by a length of very light string) that swings with a period of 1.0 second. What do you need to do to increase the period to 4.0 seconds?

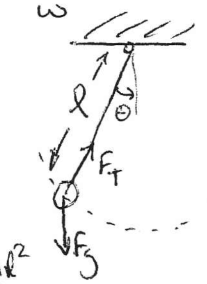
a) Decrease the length by 16.
b) Decrease the length by 4.
c) Decrease the mass by 4.
d) Increase the mass by 4.
e) Increase the length by 4.
f) Increase the length by 16.
g) Increase the length by 4 and mass by 2.

$T = \frac{1}{f}$ $\omega = 2\pi f$ so $T = \frac{2\pi}{\omega}$

Since needs to be longer by square of the increase

$\omega_0 = \sqrt{\frac{g}{\ell}}$ $\therefore T = 2\pi\sqrt{\frac{\ell}{g}}$

NOT mass dependent



FROM $\Sigma \tau = I\alpha$ about support

$F_g \times \ell = mg\ell\sin\theta = I\alpha$ $I = m\ell^2$ $\alpha = \frac{d^2\theta}{dt^2}$

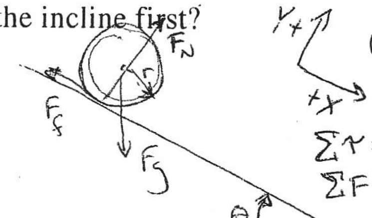
$\therefore \frac{d^2\theta}{dt^2} = \frac{g}{\ell}\theta$

$\theta = \theta_0\sin(\omega_0 t + \phi)$ $\sin\theta \sim \theta$ for small angles

The "solution" is determined by noting that this is a SHO (oscillator) so $\omega_0 = \sqrt{\frac{g}{\ell}}$ (sinusoidal function)

8) [4 pts] You decide to roll an empty and filled soup can down an incline. Both cans roll without slipping. The empty soup can is much larger and hence has the same moment of inertia as the smaller filled soup can. Which can reaches the bottom of the incline first?

a) The empty soup can reaches the bottom first.
b) Both soup cans reach the bottom at the same time.
c) The filled soup can reaches the bottom first.
d) Depends on the respective diameters of each can.



$+y$ $+x$ $\otimes +$ [for torque]

$\Sigma\tau = I\alpha$

$\Sigma F = ma$ $a = r\alpha$

$a_D > a_R$

Does not matter mass or radius just the mass distribution

$a = \frac{g\sin\theta}{(1+k)}$

$k_{Disk} = \frac{1}{2}$ $k_{ring} = 1$

$rF_f = I\alpha = kmr^2\left(\frac{a}{r}\right)$

$F_f = kma$

$F_g\sin\theta - F_f = ma$

Do two out of the next three problems. Clearly indicate which two problems you would graded.

9) [12 pts] A 0.50 kg mass is attached to the bottom of a spring attached to a stand. The system is started oscillating by displacing the mass from its equilibrium position. The resulting motion of the mass is described in SI units by the function $y(t) = 0.10\sin(10t - 1.57)$.

a) What is the spring constant of the spring?
b) What is the maximum energy stored in the spring?
c) What is the maximum kinetic energy of the mass?

10) [12 pts] Consider a planet with ½ the radius and 4 times the mass of the earth.

a) What is the orbital velocity for a 2140 kg satellite in a circular orbit at 4 times the planet's radius (or $2R_E$)?
b) Compare the energy needed to launch the satellite in to outer space to the energy required to launch the satellite into the orbit above. NOTE: Don't forget gravitational potential energy.

11) [12 pts] You need to push a heavy box across a ballroom. The kinetic coefficient of friction is 80% the static coefficient of friction, $\mu_k = 0.8\mu_s$.

a) What is the minimum force needed to start the box sliding?
b) If you keep pushing the box with the same force (from above), what is the velocity of the box at any time? NOTE: Assume you have enough floor space so you won't run into a wall anytime soon.

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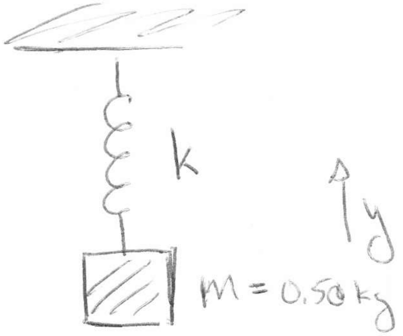

$$y(t) = 0.10 \sin(10t - 1.57)$$
$$= y_0 \sin(\omega_0 t - \phi)$$

4 pt (a) $\omega_0 = \sqrt{\frac{k}{m}}$ $\quad k = \omega_0^2 m$

$$= \left(10 \frac{\text{rad}}{s}\right)^2 \cdot 0.50\,\text{kg} = 50 \frac{\text{kg}}{s^2}$$

4 pt (b) Energy stored in the spring $\quad \int F \cdot dx = \int \frac{d}{8} kx\, dx = \frac{1}{2} kx^2$

- where x is the displacement

$E_{max} = \frac{1}{2} k (y_{max})^2 \quad y_{max} = 0.10\,\text{m}$

$$= \frac{1}{2}\, 50 \frac{\text{kg}}{s^2} \cdot (0.10\,\text{m})^2 = 0.25\ \text{J}$$

4 pt (c) Maximum kinetic energy is when mass is moving the fastest

$v(t) = \frac{d}{dt} y(t) = (y_0 \omega_0) \cos(\omega_0 t - \phi)$

max velocity

$KE_{max} = \frac{1}{2} m v_{max}^2$

$= \frac{1}{2} (0.5\,\text{kg})(1\,\text{m/s})^2 \qquad v_{max} = (0.10\,\text{m})\left(10 \frac{\text{rad}}{s}\right) = 1\,\text{m/s}$

$= 0.25\ \text{J}$

This is reasonable since you expect energy to be conserved so max spring energy is swapped to max kinetic energy

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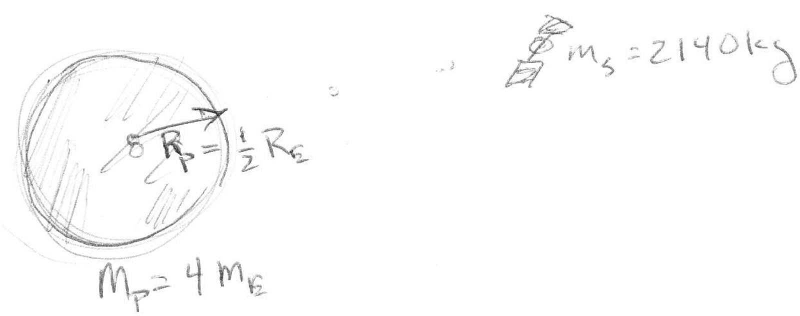
$\boxed{10}$



(a) Determine orbital velocity

$$\vec{v} \qquad F_g = \frac{G M_p m_s}{r^2}$$

$$V_{orb} = \left(\frac{G M_p}{r}\right)^{1/2} \qquad r = 4R_p = 2R_E \qquad M_p = 4M_E$$

$$\Sigma F = ma$$

$$\frac{G M_p m_s}{r^2} = F_g = m_s a_c = m_s \frac{V^2}{r}$$

m_s cancels

$$= \left[\frac{G(4M_E)}{(2R_E)}\right]^{1/2} = \left[2R_E \frac{G M_E}{R_E^2}\right]^{1/2} \qquad \frac{G M_E}{R_E^2} = g$$

$$V_{orb} = (2 R_E g)^{1/2} = \left(2 \cdot 6.38\times10^6 m \cdot 9.81 \frac{m}{s^2}\right)^{1/2}$$

$$= 1.12\times10^4\ m/s \qquad \text{NOTE: } V_{orb}^2 = \frac{G M_p}{4R_p}$$

* same as escape velocity from surface of earth

(b) Energy needed to escape into outer space –

$$E_i = E_f$$

$$-\frac{G M_p m_s}{R_p} + \frac{1}{2} m_s V_{esc}^2 = 0 + 0 \qquad V_{esc} = \left(\frac{2 G M_p}{R_p}\right)^{1/2} = \left(\frac{2G M_E \cdot 4}{\frac{1}{2}R_E}\right)^{1/2} = \sqrt{8}\, V_{esc\ earth}$$

This is the energy required

$$E_{esc} = \frac{G M_p m_s}{R_p} \qquad \text{NOTE } E_{esc} = \frac{G(4M_E)}{\frac{1}{2}R_E} m_s = 8\frac{G M_E}{R_E} m_s = \boxed{1.07\times10^{12} J}$$

Energy needed to put satellite into orbit –

$$E_i = E_f$$

$$-\frac{G M_p m_s}{R_p} + \frac{1}{2} m_s V^2 = \frac{1}{2} m_s V_{orb}^2 - \frac{G M_p m_s}{r} \qquad r = 4R_p$$

This is the energy required

$$E_{orb} = \frac{1}{2} m_s V_{orb}^2 + G M_p m_s\left(\frac{1}{R_p} - \frac{1}{4R_p}\right) = \frac{1}{2} m_s V_{orb}^2 + \frac{G M_p m_s}{R_p}\underbrace{\left(1 - \frac{1}{4}\right)}_{3/4} = \frac{G M_p m_s}{R_p}\left(\frac{7}{8}\right)$$

$$\frac{1}{2} m_s \frac{G M_p}{4R_p} = \frac{1}{8}\frac{G M_p m_s}{R_p}$$

$$\boxed{E_{esc} > E_{orb} = \frac{7}{8} E_{esc}} = \boxed{0.94\times10^{12} J}$$

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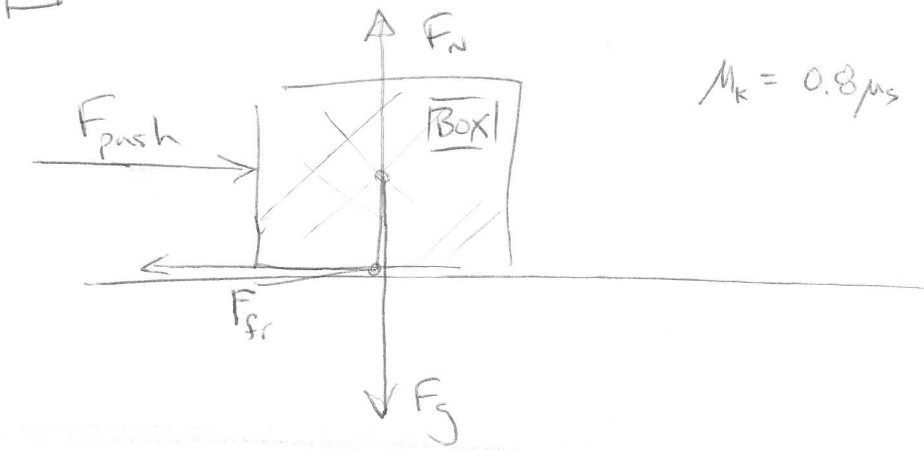
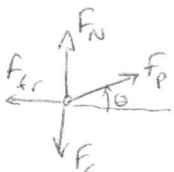
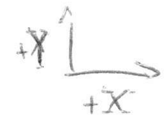


$\mu_k = 0.8\mu_s$

(a) Find minimum force needed to get box sliding



$\sum F = ma$



$Y: F_N + F_p \sin\theta - F_g = 0 \qquad F_N = F_g - F_p \sin\theta$

$X: F_p \cos\theta - F_{fr} = m a_y \qquad F_{fr} = \mu_s F_N$

for minimum force let $F_p \cos\theta \geq F_{fr}$ set $a_y \approx 0$

$F_p \cos\theta \geq \mu_s (F_g - F_p \sin\theta)$ solve for F_p

$F_p(\cos\theta + \mu_s \sin\theta) \geq \mu_s mg$

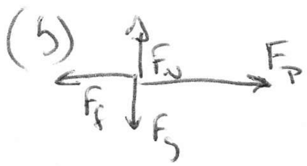
$$F_p \geq \frac{\mu_s mg}{(\cos\theta + \mu_s \sin\theta)}$$

*Assume $\theta = 0°$

NOTE: Minimum force depends on μ_s and θ.

$F_p \geq \mu_s mg$

(b)



$F_N + F_p \sin\theta - F_g = 0 \qquad \theta = 0 \qquad F_N = F_g \therefore \mu F_N = F_f$

$F_p \cos\theta - F_{fr} = m a_y$

$F_p - \mu_k F_g = m a_y \qquad F_p = \mu_s mg$

$\mu_s mg - \mu_k mg = m a_y \qquad a_y = g(\mu_s - \mu_k) = 0.2\mu_s g$

$v(t) = \int a_y \, dt = 0.2\mu_s g t + v_0 \qquad v_0 = 0\,\text{m/s} \therefore v(t) = 0.2\mu_s g t$